TMA-101 | Printed Pages : 5

Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

(Sem. I) UTU EXAMINATION, 2013-14

Mathematics

Time: Three Hours | *Maximum Marks : 100*

Note: Attempt all questions, the marks assigned to each question is indicated at question itself.

1. Attempt any four (5x4)

   a. Show that the vectors $X_1 = (1,1,2)$, $X_2 = (1,2,5)$ and $X_3 = (5,3,4)$ are linearly dependent. Also express each vector as a linear combination of the other two.

   b. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$. Hence find A^3.

   c. Test whether the system of equations is consistent. $x + 2y + 3z = 0$, $3x + 4y + 4z = 0$, $7x + 10y + 11z = 0$. Hence find the solution if it exist.

d. Find the eigen values of the matrix

$$A=\begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}.$$

e. Find the eigen vectors corresponding to eigen values 5,1,1 of a matrix $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

f. Is the matrix $A=\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ is diagonalizable? Justify your answer.

2. Attempt any four. (5x4)

a. Show that

$$\frac{d}{da}\int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right)dx = 2a\tan^{-1}a-\frac{1}{2}\log\left(a^2+1\right)$$

b. If $z=e^{ax+by} f(ax-by)$, prove that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=abz.$

contained in the first octant between the planes z=0 and z = 2.

(ii) Find the value of λ, so that $F = \lambda y^4 z^2 i + 4x^3 z^2 j + 5x^2 y^2 k$ may be solenoidal. [7 +3]

b. (i) Use Green's theorem in a plane to find the finite area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

(ii) Using Gauss divergence theorem, evaluate $\iint_S x^3\, dydz + x^2 y\, dzdx + x^2 z\, dx\, dy$, where S: closed surface consisting of the circular cylinder $x^2 + y^2 = a^2, (0 \le z \le b)$ and the circular disks z=0 and z=b. $\left(x^2 + y^2 \le a^2\right)$ [4+6]

c. Verify Stoke's theorem for $\vec{F} = -y\hat{i} + 2yz\hat{j} + y^2\hat{k}$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and C is the circular boundary on the xoy plane. [10]

c. Verify Euler's theorem for a function $f(x, y, z) = 3x^2 yz + 5xy^2 z + 4z^4$

d. Find the Taylor's series expansion of x^y near the point (1,1) upto the second degree terms.

e. if $z = \log\left(u^2 + v\right), u = e^{x^2 + y^2}, v = x^2 + y$, find $\frac{\delta z}{\delta u}, \frac{\delta z}{\delta y}$.

f. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, find $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$.

3. Attempt any two (10x2)

a. Examine the extrema of $f(x,y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$ [10]

b. Find the volume of the greatest rectangular parallelopiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ [10]

c. (i) Are these functions $f_1 = x + y + z$, $f_2 = x^2 + y^2 + z^2$ and $f_3 = xy + yz + xz$ functionally dependent? If so, find the relation among f_1, f_2 and f_3.

(ii) The time of oscillation of a simple pendulum is given by the equation $T = 2\pi\sqrt{l/g}$. In an experiment carried out to find the value of g, errors of 1.5% are possible in the values of l and T respectively. Show that the error in the calculated value of g is 0.5%.

[5+5]

4. Attempt any two. (10x2)

a. (i) Change the order of integration in $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{y^2}^{4-y^2} x\,dxdy$ and then evaluate.

(ii) Transform the double integral

$$\int_0^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$$ in to polar coordinates and then evaluate it. [5+5]

b. (1) Find the area that lies inside the cardioid $r = (1+\cos\theta)$ and outside the circle $r = a$, by double integration.

(2) Prove that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}} dx \times \int_0^\infty x^2 e^{-x^4} dx = \frac{\pi}{4\sqrt{2}}$

[6+4]

c. (i) Evaluate $\iiint_V xyz\,dx\,dy$ where V is the region of space inside the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = 1$.

(ii) Show that $\iint x^{m-1} y^{n-1}\,dxdy$ over the positive quadrant of the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ is

$$\frac{a^m b^n}{2n} \beta\left(\frac{m}{2}, \frac{n}{2} + 1\right)$$ [6+4]

5. Attempt any two. (10x2)

a. (i) Evaluate $\iint_S \vec{F}.dS$, if $\vec{F} = yz\hat{i} + 2y^2\hat{j} + xz^2\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$