(b) A homogeneous rod of conducting material of length 'l' has its ends kept at zero temperature. The temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature distribution.

(c) Solve $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given $u(0,y) = 4e^{-y} - e^{-5y}$, by the method of separation of variables.

PAPER ID : 9917

TMA-201 Printed Pages : 4

Paper ID and Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B. Tech.

(SEM. II) EXAMINATION, 2012

MATHEMATICS - II

Time : 3 Hours] [Total Marks : 100

Note : Attempt all questions. All questions carry equal marks.

1 Attempt any four parts of the following : 5×4=20

(a) Solve :

$$\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$$

(b) Solve :

$$\left(1+y^2\right)dx=\left(\tan^{-1}y-x\right)dy$$

(c) Solve :

$$\left(D^2-2D+1\right)y=x\sin x$$

(d) $\frac{dx}{dt}+2x-3y=t,\ \frac{dy}{dt}-3x+2y=e^{2t}$

(e) Solve :

$$\frac{d^2y}{dx^2}+\cot x\frac{dy}{dx}+4y\,\mathrm{cosec}^2x=0$$

by changing the independent variable.

(f) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2}+n^2=\sec nx$$

2 Attempt any two parts of the following : **10×2=20**

(a) Find Laplace Transform of

(i) $te^{at}\sin at$

(ii) $(\cos at - \cos bt)/t$

(b) (i) Find $L^{-1}\left[\log\frac{S+1}{S-1}\right]$

(ii) Find $L^{-1}\left[\frac{1}{S(s+a)^3}\right]$

(c) Using Laplace Transform solve the following equations :

$$\frac{d^2x}{dt^2}+x=t\cos 2t$$

Given $x(0) = x'(0) = 0$

3 Attempt any two parts of the following : **10×2=20**

(a) (i) Test for convergence the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+-----\infty$$

(ii) Test for convergence the series

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}-----\infty$$

(b) Discuss the convergence of the series

$$\sum_{2}^{\infty}\frac{1}{n(\log n)^p};\ p>0,$$ by Integral Test.

(c) Show that the Geometric series $\sum_{n=0}^{\infty} r^n$, $(r>0)$ is convergent when $r<1$ and divergent when $r\geq 1$

4 Attempt any two parts of the following :

(a) Find the Fourier series expansion for f(x), if

$$f(x)\begin{cases}-\pi, -\pi<x<0\\ x, 0<x<\pi\end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+---=\frac{\pi^2}{8}$

(b) Expand $f(x)=\begin{cases}\frac{1}{4}x, 0<x<1/2\\ x-3/4, 1/2<x<1\end{cases}$

as the Fourier sine series

(c) Solve $(D^3-4D^2D' + 4DD'^2)z = 6\sin(3x+2y)$

5 Attempt any two parts of the following : **10×2=20**

(a) A tightly stretched string with fixed end points x=0 nd x=1 is initially in a position given by $y(x,0) = y_0\sin(\pi x/l)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.