Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B.Tech

SEMESTER-III, UTU EXAMINATION, 2013-14

Discrete Structure

COMPUTER SCIENCE & ENGINEERING

Time: **3** Hours] [Total Marks: **100**

Q1: Attempt any four of the following **4 x 5 = 20**

1. Prove that the complement of the *union of two sets is* the intersection of their complements?
2. What is meant by composition of two functions? Let f: R □ R. and g: R □ R, where R is the set of real numbers. Find fog and gof, where $f(x) = x^2 - 2$ and $g(x) = x + 4$.
3. *What is an* equivalence relation? Show that the relation of 'similarity' on the set of all triangles in a plane is an equivalent relation?
4. Define symmetric difference and *disjoint* set.
5. Let N be the set of all natural numbers. The relation R on set NxN is defined as: (a, b) R (c, d) ↔ ad = bc
   Prove that R is an equivalence relation.

Q2: Attempt any four of the following **4 x 5 = 20**

1. If (S, *) be the commutative *semigroup. Show that if a*a = a,* and b*b = b, *then* (a*b)*(a*b) = a*b.
2. Show that the set, $G = \{1, w, w^2\}$, where 1, w, w^2 are the cube roots of the unity, forms an Abelian group under the operation of ordinary multiplication.

3. Define a subgroup. When a subgroup is said to be normal? Explain with example.
4. Show that the set of N natural numbers is a semigroup under the operation x * y = max(x, y). Is it a monoid?
5. Define permutation group. Let A = {1, 2, 3, 4, 5}. Find (1 3) 0 (2 4 5) 0 (2 3).

Q3: Attempt any FOUR of the following **4 x 5 = 20**

1. Let A = {1, 2, 3, 4} and consider the relation
   R = {(1, 1), (2, 1), (2, 2), (3, 1), (3, 3), (3, 4), (4, 4)}
   Show that R is a partial ordering, and draw its Hasse diagram.
2. Prove that a finite partial ordered set has
   a. At most one greatest element.
   b. At most one least element.
3. Consider the subsets {2, 3}, {4, 6} and {3, 6} in the poset ({1, 2, 3, 4, 5, 6}, 1), find for each subsets, if exists
   a. Upper bound and lower bound
   b. Greatest lower bound and least upper bound
4. The set P({a, b, c}) is partially ordered with respect to the subset relation. Find a chain of length 3 in P({a, b, c}).
5. Define different types of lattices.

Q4: Attempt any TWO of the following **2 x 10 = 20**

1. Express the following statements using predicates and quantifiers:
   (a) For every student in this class, that student has studied mathematics.
   (b) Some students in this class have visited United States.
   (c) Every student in this class visited either Canada or United States.

2. **(a)** Show that [(P ∨ ~Q) ∧ (~P □ ~Q)] ∨ Q is a tautology.
   (b) Show that (~P ∧Q) □ (Q□P) is not a tautology.
3. **(a)** Show that (P □C) ∧ (Q □C) is logically equivalent to (P ∨ Q) □ C
   (b) Differentiate between tautology and contradiction.

Q5: Attempt any TWO of the following **2 x 10 = 20**

1. Find the solution of the following recurrence relation relations with the initial conditions given:
   a. $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with $a_0 = 2$, $a_1 = 5$, $a_2 = 15$.
   b. $a_n = 3a_{n-1} + 2n$ with $a_1 = 3$.
2. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if,
   a. They can be any color.
   b. Two must be white and two must be red.
   c. They must all be of the same color.
3. Let H_n denote the number of moves needed to solve the Tower of Hanoi problem for n disks with initial condition as $H_1 = 1$. Set up a recurrence relation for the sequence $\{H_n\}$