

(d) Find the Laurent's expansion for : 5

$$f(z) = \frac{7z-2}{z^3 - z^2 - 2z}$$

in the regions given by $1 < |z+1| < 3$

(e) Determine the poles of the following function 5
and residues at each pole :

$$f(z) = \frac{z-1}{(z+1)^2(z-2)} \text{ and hence evaluate}$$

$$\oint_c f(z) dz, \text{ where } c \text{ is a circle } |z-i|=2.$$

(f) Apply calculus of residues to prove that 5

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{(a+b)} \quad (a > 0, b > 0)$$

2 Attempt any two questions out of following :

(a) (i) From the following data, calculate the 5
first four moments about the mean :

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	8	11	15	11	7	4

(ii) The first four moments about the 5
working mean 28.5 of a distribution are
0.294, 7.144, 42.409 and 454.98. Calculate
the moment about mean. Also, evaluate
 β_1 , β_2 and comment upon the skewness
and kurtosis of the distribution.

(c) (i) Solve $\frac{dy}{dx} = yz + x$ and $\frac{dz}{dx} = xz + y$ given 5
that $y(0) = 1, z(0) = -1$ for $y(0.1), z(0.1)$.

(ii) Use Gauss-Seidel iterative method to
solve the following system of equations : 5

$$9x + 4y + z = -17$$

$$x - 2y - 6z = 14$$

$$x + 6y = 4$$

(ii) Using Lagrange's interpolation formula, find $y'(10)$ from the following table :

x	5	6	9	11
y	12	13	14	16

5 Attempt any two parts :
 (a) (i) Use Gauss elimination method to solve the following system of equations :

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

$$x + 4y + 9z = 16$$

(ii) Find $\frac{dy}{dx}$ at $x=0.1$ from the following table :

x	0.1	0.2	0.3	0.4
y	0.9975	0.9900	0.9776	0.9604

(b) (i) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{4}$.

(ii) Find the solution of $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$ which passes through (0, 1) in the interval (0, 0.5) such that the value of y is correct to three decimal places. Take $h = 0.1$.

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(ii) Five defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

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3 Attempt any two parts out of following :

- (a) (i) A manufacturer knows that the condensers he makes contains on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensers ?
- (ii) Intelligence test of two groups of boys and girls gave the following results :

	Mean	S.D.	Size
Girls	84	10	121
Boys	81	12	81

- Is the diff. in mean scores significant ?
Also test for S.D.
- (b) (i) The following frequency distribution gives the frequencies of seeds in a pea breeding experiment :

Round and yellow	Wrinkled and yellow	Round and green	Wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be 9:3:3:1. Examine the correspondance between theory and experiment.

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(ii) Fit a trend line using free hand graphical method :

Years	1980	1981	1982	1983	1984	1985	1986
Sales	35	60	45	70	65	62	80

(c) (i) It is desired to compare three hospitals with regards to the number of deaths per month. A sample of death records were selected from the records of each hospital and the number of deaths were given below. From these data, suggest the difference in the number of the deaths per months among three hospitals :

Hospitals	A	B	C
	3	6	7
	4	3	3
	3	3	4
	5	4	6
	0	4	5

(ii) Before an increase in excise duty on tea, 800 people out of 1000 persons were found to be tea drinkers. After an increase in duty 800 persons were known to be tea drinkers in a sample of 1200. Do you think that there has been significant decrease in the consumption of tea after the increase in excise duty ?

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(b) (i) Obtain the least square fit of the form $f(t) = a e^{-3t} + b e^{-2t}$ for the data

t :	0.1	0.2	0.3	0.4
$f(t)$:	0.76	0.58	0.44	0.35

(ii) The following gives frequency of marks obtained by 100 students in an intelligence test according to the age.

Age in yrs.	18	19	20	21	Total
Marks					
10-20	4	2	2		8
20-30	5	4	6	4	19
30-40	6	8	10	11	35
40-50	4	4	6	8	22
50-60		2	4	4	10
60-70		2	3	1	6
Total	19	22	31	28	100

Calculate the coefficient of correlation between age and intelligence.

(c) (i) The equations of two regression lines, obtained in a correlation analysis of 60 observations are

$$5x = 6y + 24 \text{ and } 1000y = 768x - 3608$$

What is the correlation coefficient? Show that the ratio of coefficient of variability of x to y is $\frac{5}{24}$. What is the ratio of variances of x and y ?

4 Attempt any two parts : 5

(a) (i) Apply false position method to find positive root of the equation $x - e^{-x} = 0$ correct to three decimal places.

(ii) Determine the value of p and q so that rate of convergence of the iterative method

$$x_{n+1} = + p x_n + q \frac{N}{x_n^2}$$

for computing $N^{1/3}$ becomes as high as possible.

(b) (i) Estimate the production of cotton in the year 1935 from the data given below :

year (x)	1931	1932	1933	1934	1935	1936	1937
Production	17.1	13	14	9.6	-	12.4	18.2
$f(x)$							

(ii) Express $y = 2x^3 - 3x^2 + 3x - 10$ in factorial notation and hence show that $\Delta^3 y = 12$.

(c) (i) Evaluate $\Delta^2 \left(\frac{5x+12}{x^2+5x+16} \right)$; the interval of differencing being unity.